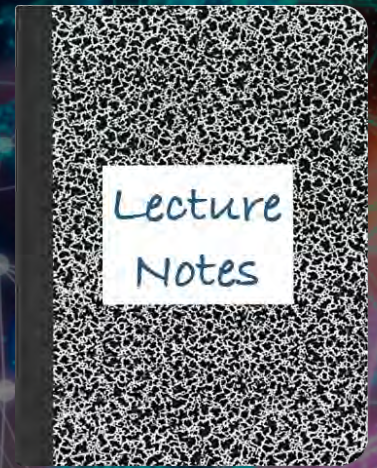


CS 419: Computer Security

# Week 3: Asymmetric Cryptography & Integrity

Paul Krzyzanowski

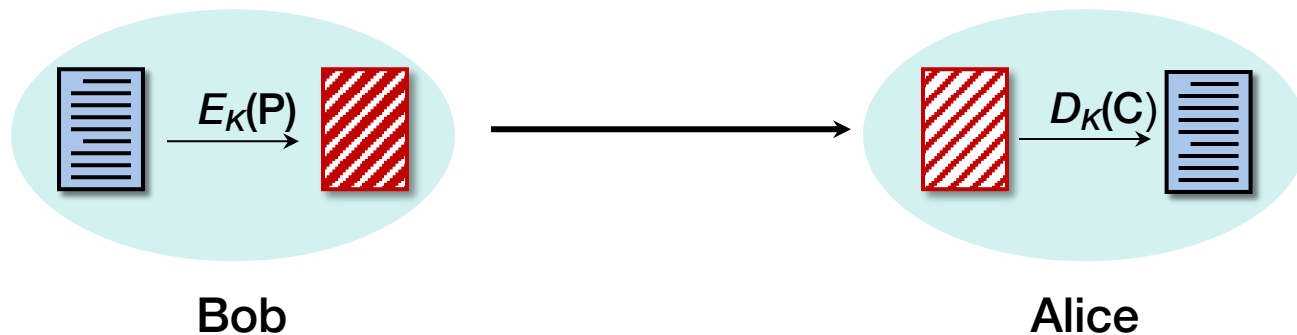


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# Key Distribution

# Communicating with symmetric cryptography

- Both parties must agree on a secret key,  $K$
- Message is encrypted, sent, decrypted at other side



**Key distribution must be secret. Otherwise**

- Messages can be decrypted by the adversary
- Users can be impersonated

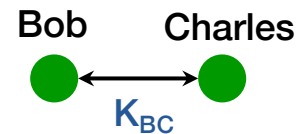
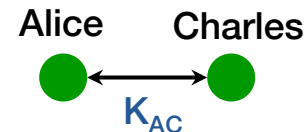
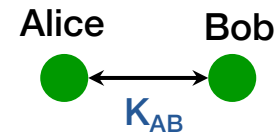
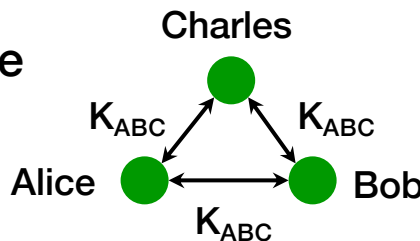
# Problems With Keys In Symmetric Cryptography

## Key Management

- Potentially a lot of keys to track
- Every communicating group of users needs a key

## Key Distribution

- How do you communicate with someone you've never met?
- You cannot send them the secret key if the communication line is not secure



**Secure key distribution is the biggest problem with symmetric cryptography**

# Public Key Cryptography

# Public-key algorithm

Two related keys:

$$C = E_{K_1}(P) \quad P = D_{K_2}(C)$$

$$C' = E_{K_2}(P) \quad P = D_{K_1}(C')$$

$K_1$  is a **public** key

$K_2$  is a **private** key

**Examples:**

RSA, Elliptic curve algorithms

DSS (digital signature standard)

# Trapdoor functions

Public key cryptography relies on **trapdoor functions**

## Trapdoor function

- Easy to compute in one direction
- The inverse is difficult to compute without extra information

## Example:

**96171919154952919** is the product of two prime #s. What are they?

But if you're told that one of them is **100225441**

... then it's easy to compute the other: **959555959**



# RSA Public Key Cryptography

**Ron Rivest, Adi Shamir, Leonard Adleman created the first public key encryption algorithm in 1977**

**Each user generates two keys:**

**Private key** (kept secret)

**Public key** (can be shared with anyone)

**Difficulty of algorithm based on the difficulty of factoring large numbers**

Keys are functions of a pair of large (~300 digits) prime numbers

# RSA algorithm: key generation

1. Choose two random large prime numbers  $p, q$
2. Compute the product  $n = pq$  and  $\phi(n) = (p - 1)(q - 1)$   
 $n$  will be presented with the public & private keys. Length( $n$ ) is the **key length**
3. Choose the **public exponent**,  $e$ , such that:  
 $1 < e < \phi(n)$  and  $\gcd(e, \phi(n)) = 1$  [  $e$  and  $(p - 1)(q - 1)$  are relatively prime ]
4. Compute the **secret exponent**,  $d$  such that:  
 $ed = 1 \pmod{\phi(n)}$   
 $d = e^{-1} \pmod{(p - 1)(q - 1)}$
5. **Public key** =  $(e, n)$   
**Private key** =  $(d, n)$   
Discard  $p, q, \phi(n)$

See [https://www.di-mgt.com.au/rsa\\_alg.html](https://www.di-mgt.com.au/rsa_alg.html)

# RSA Encryption

**Key pair:** public key =  $(e, n)$   
private key =  $(d, n)$

## Encrypt

- Divide data into numerical blocks  $< n$
- Encrypt each block:

$$c = m^e \bmod n$$

## Decrypt

$$m = c^d \bmod n$$

# RSA security

The security of RSA encryption rests on the difficulty of factoring a large integer

**Public key** = { *modulus*, *exponent* }, or { *n*, *e* }

- The *modulus* is the product of two primes, *p*, *q*
- The private key is derived from the same two primes

# Weak RSA Public Keys

March 14, 2022

- Older software generated RSA keys that can be broken instantly with commodity hardware
- SafeZone library doesn't randomize the prime numbers well
  - Used to generate RSA keys
  - After selecting one prime #, the second one is in close proximity to the first
- Keys generated with primes that are too close together can be broken with Fermat's factorization method, described in 1643

<https://arstechnica.com/information-technology/2022/03/researcher-uses-600-year-old-algorithm-to-crack-crypto-keys-found-in-the-wild/>

**ars TECHNICA** SUBSCRIBE SEARCH SIGN IN

**BREAKING KEYS —**  
**Researcher uses 379-year-old algorithm to crack crypto keys found in the wild**

It takes only a second to crack the handful of weak keys. Are there more out there?

DAN GOODIN - 3/14/2022, 5:31 PM



**Enlarge**

Cryptographic keys generated with older software now owned by technology company Rambus are weak enough to be broken instantly using commodity hardware, a researcher reported on Monday. This revelation is part of an investigation that also uncovered a handful of weak keys in the wild.

The software comes from a basic version of the SafeZone Crypto Libraries, which were developed by a company called Inside Secure and acquired by Rambus as part of its 2019 acquisition of Verimatrix, a Rambus representative said. That version was deprecated prior to the acquisition and is distinct from a FIPS-certified version that the company now sells under the Rambus FIPS Security Toolkit brand.

# Weak RSA Public Keys

- **Product of two large primes can be written as**  
$$N = (a-b)(a+b)$$
  - where  $a$  is the middle between the two primes
  - $b$  is the distance from the middle to each of the primes
- **If the primes are close, then  $a$  is close to  $\sqrt{N}$**
- **Attack: guess  $a$  by starting from  $\sqrt{N}$  and then incrementing the guess**
  - Calculate  $b^2 = a^2 - N$
  - If the result is a square then we guessed correctly
  - Calculate the factors  $p, q$  as  $p=a+b, q=a-b$

# Elliptic Curve Cryptography

## Elliptic curves

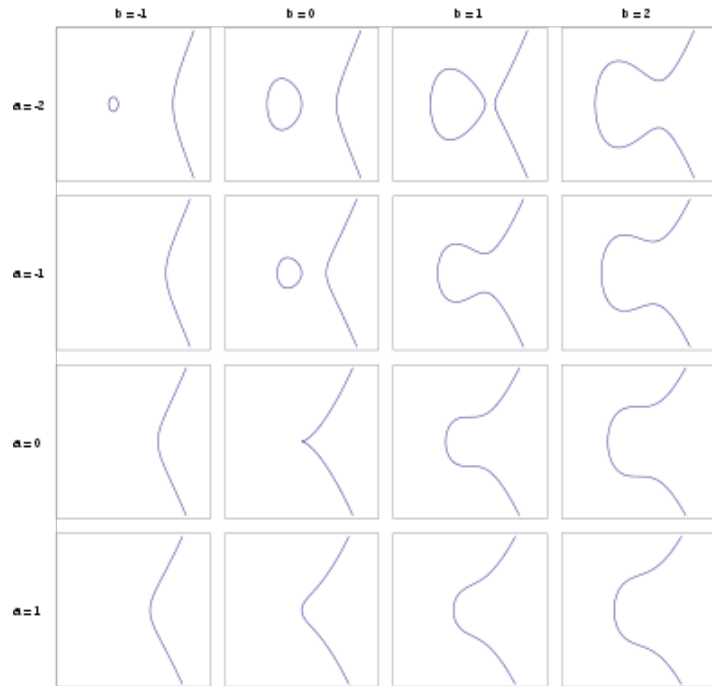
$$y^2 = x^3 + ax + b \pmod{p}$$

## Using discrete numbers, pick

- A prime number as a maximum (modulus)
- A curve equation
- A pre-defined base point on the curve (generator, G)
- A random private key, k
- Public key is derived from the private key, the base point, and the curve,  $P = k * G$ 
  - This is an efficient point multiplication process

## To compute the private key from the public,

- We would need an elliptic curve **discrete logarithm** function
- This is difficult and is the basis for ECC's security



Catalog of elliptic curves  
[https://en.wikipedia.org/wiki/Elliptic\\_curve](https://en.wikipedia.org/wiki/Elliptic_curve)

See <https://cryptobook.nakov.com/asymmetric-key-ciphers/elliptic-curve-cryptography-ecc#>

Also <https://blog.cloudflare.com/a-relatively-easy-to-understand-primer-on-elliptic-curve-cryptography>

# ECC vs. RSA

- **RSA is still a widely used public key cryptosystem (but fading)**
  - Mostly due to inertia & widespread implementations – it had a 27-year head start
  - Trusted, well-tested deployments
  - Trust in the algorithm  
(there was initial skepticism over the choice of curves and trust in the NIST, who approved them; the NSA tried to push an insecure random number generator)
  - Simpler implementation
- **ECC offers higher security with fewer bits than RSA**
  - ECC is faster for key generation & encryption
    - The private key is any random number within a certain range (e.g., a 512-bit integer)
    - Encryption is about 10x faster than RSA
  - Uses less memory
  - NIST defines 15 standard curves for ECC
    - But many implementations support only a couple (P-256, P-384)

<https://www.keylength.com/en/4/>

<http://https://www.enisa.europa.eu/publications/algorithms-key-size-and-parameters-report-2014>



# Key length

**Unlike symmetric cryptography, not every number is a valid key with RSA**

**Comparable complexity:**

- 3072-bit RSA = 256-bit elliptic curve = 128-bit symmetric cipher
- 15360-bit RSA = 512-bit elliptic curve = 256-bit symmetric cipher

**For long-term security**

The European Union Agency for Network and Information Security (ENISA) and the National Institute for Science & Technology (NIST) recommend:

AES: 256-bit keys

RSA: 15,360-bit keys

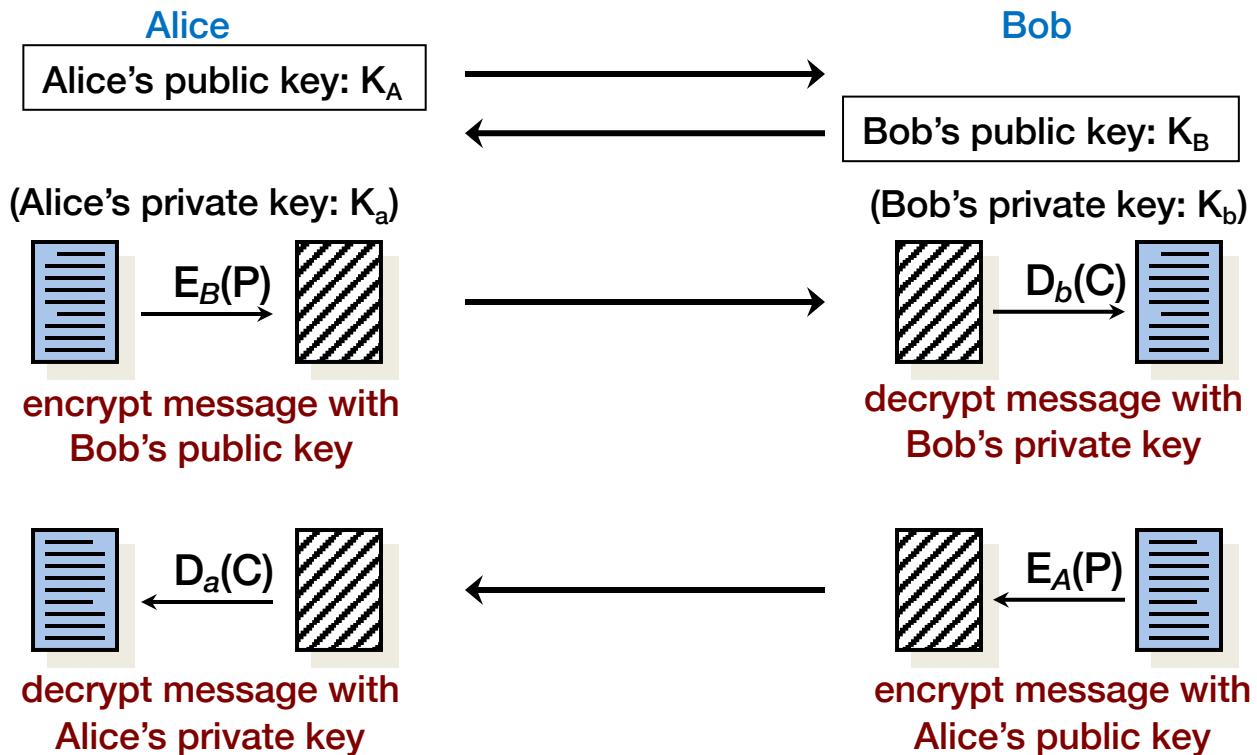
ECC: 512 bit-keys

# Communication with public key algorithms

## **Different keys for encrypting and decrypting**

- No need to worry about key distribution

# Communication with public key algorithms



# RSA isn't good for communication

Calculations are very expensive relative to symmetric algorithms

Common speeds:

Algorithm	Bytes/sec
AES-128-ECB	148,000,000
AES-128-CBC	153,000,000
AES-256-ECB	114,240,000
RSA-2048 encrypt	3,800,000
RSA-2048 decrypt	96,000

**AES ~1500x faster to decrypt; 40x faster to encrypt than RSA**

# Public key algorithms are not used for communication

- **Vulnerability to known plaintext attacks (or guessing)**
  - Content must be broken into smaller blocks since each block is treated like a number. An attacker can encrypt a wide set of predicted content with the recipient's public key and then look for matches in the ciphertext.
    - If I send "Yes" to you, I need to encrypt it with your public key. The attacker can encrypt "Yes", "No", and any other expected content with that same public key and see what matches the content I send. If there's a predicted chunk of a message, the attacker can spot it.
- **Some algebraic relationships may be preserved**
  - Some algebraic relationships that exist between plaintext content may exist with public key algorithms. This can provide an attacker with insights on the relationship between content
- **ECC is faster than RSA and uses shorter keys**
  - Still slower than symmetric algorithms
  - ECC public key generation is efficient compared with RSA but still requires math (point multiplication): see [https://en.wikipedia.org/wiki/Elliptic\\_curve\\_point\\_multiplication](https://en.wikipedia.org/wiki/Elliptic_curve_point_multiplication)

# Key Exchange

# Diffie-Hellman Key Exchange

## Key distribution algorithm

- Allows two parties to share a secret key over a non-secure channel
- Not public key encryption
- Based on difficulty of computing discrete logarithms in a finite field compared with ease of calculating exponentiation

Allows us to negotiate a secret **common key** without fear of eavesdroppers

# Diffie-Hellman Key Exchange

- All arithmetic performed in a field of integers modulo some large number
- Both parties agree on
  - a **large prime number**  $p$
  - and a **number**  $\alpha < p$
- Each party generates a public/private key pair

Private key for user  $i$ :  $X_i$

Public key for user  $i$ :  $Y_i = \alpha^{X_i} \bmod p$



# Diffie-Hellman Key Exchange

- Alice has secret key  $X_A$
- Alice sends Bob public key  $Y_A$
- Alice computes
- Bob has secret key  $X_B$
- Bob sends Alice public key  $Y_B$

$$K = Y_B^{X_A} \bmod p$$

**$K = (\text{Bob's public key}) (\text{Alice's private key}) \bmod p$**

# Diffie-Hellman Key Exchange

- Alice has secret key  $X_A$
- Alice sends Bob public key  $Y_A$
- Alice computes

$$K = Y_B^{X_A} \bmod p$$

- Bob has secret key  $X_B$
- Bob sends Alice public key  $Y_B$
- Bob computes

$$K = Y_A^{X_B} \bmod p$$

$$K' = (\text{Alice's public key}) (\text{Bob's private key}) \bmod p$$

# Diffie-Hellman Key Exchange

- Alice has secret key  $X_A$
- Alice sends Bob public key  $Y_A$
- Alice computes

$$K = Y_B^{X_A} \bmod p$$

- expanding:

$$\begin{aligned} K &= Y_B^{X_A} \bmod p \\ &= (\alpha^{X_B} \bmod p)^{X_A} \bmod p \\ &= \alpha^{X_B X_A} \bmod p \end{aligned}$$

- Bob has secret key  $X_B$
- Bob sends Alice public key  $Y_B$
- Bob computes

$$K = Y_A^{X_B} \bmod p$$

- expanding:

$$\begin{aligned} K &= Y_A^{X_B} \bmod p \\ &= (\alpha^{X_A} \bmod p)^{X_B} \bmod p \\ &= \alpha^{X_A X_B} \bmod p \end{aligned}$$

$$\mathbf{K = K'}$$

***K is a common key, known only to Bob and Alice***

# Diffie-Hellman simple example

Assume  $p=1151$ ,  $a=57$

- Alice's secret key  $X_A = 300$
- Alice's public key  $Y_A = 57^{300} \bmod p = 282$
- Alice computes
- Bob's secret key  $X_B = 25$
- Bob's public key  $Y_B = 57^{25} \bmod p = 1046$
- Bob computes

$$K = Y_B^{X_A} \bmod p = 1046^{300} \bmod p$$

$$K = 105$$

$$K = Y_A^{X_B} \bmod p = 282^{25} \bmod p$$

$$K = 105$$

*Given  $p=1151$ ,  $a=57$ ,  $Y_A=282$ ,  $Y_B=1046$ , you cannot get 105*

# Hybrid Cryptosystems

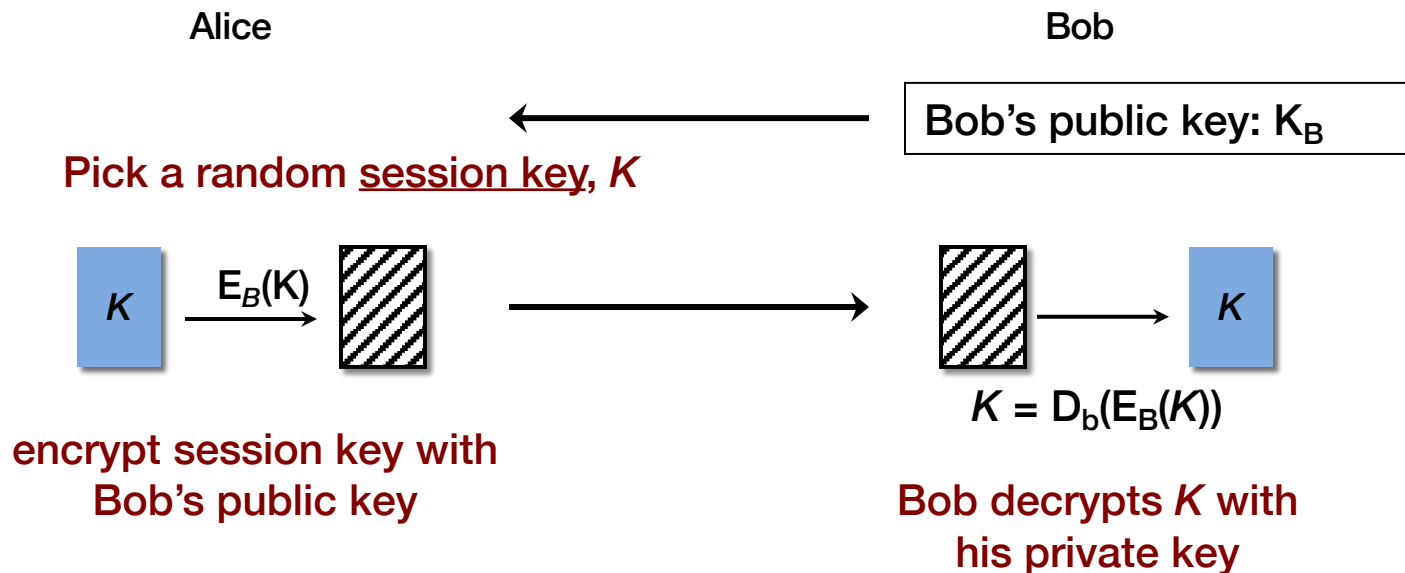
# Hybrid Cryptosystems

- **Session key**: randomly-generated key for one communication session
- Use a **public key algorithm** to send the session key
- Use a **symmetric algorithm** to encrypt data with the session key

**Public key algorithms are almost never used to encrypt messages**

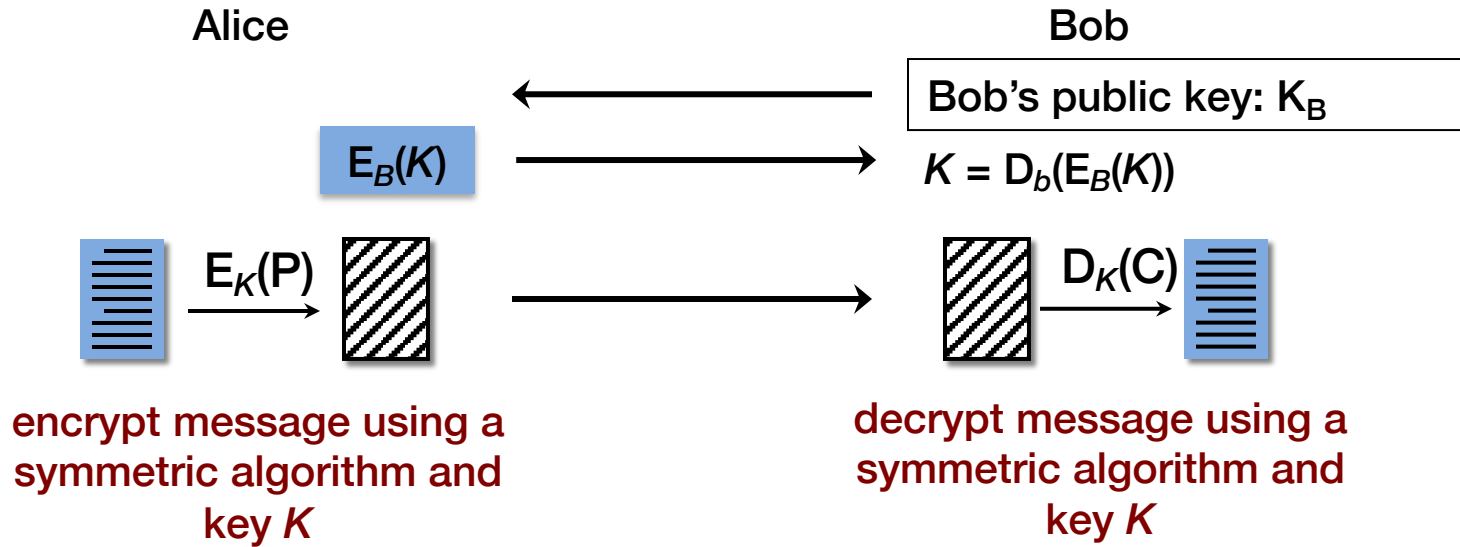
- MUCH slower; vulnerable to *chosen-plaintext* and *algebraic attacks*
- RSA-2048 approximately 55x slower to encrypt and 2,000x slower to decrypt than AES-256

# Communication with a hybrid cryptosystem



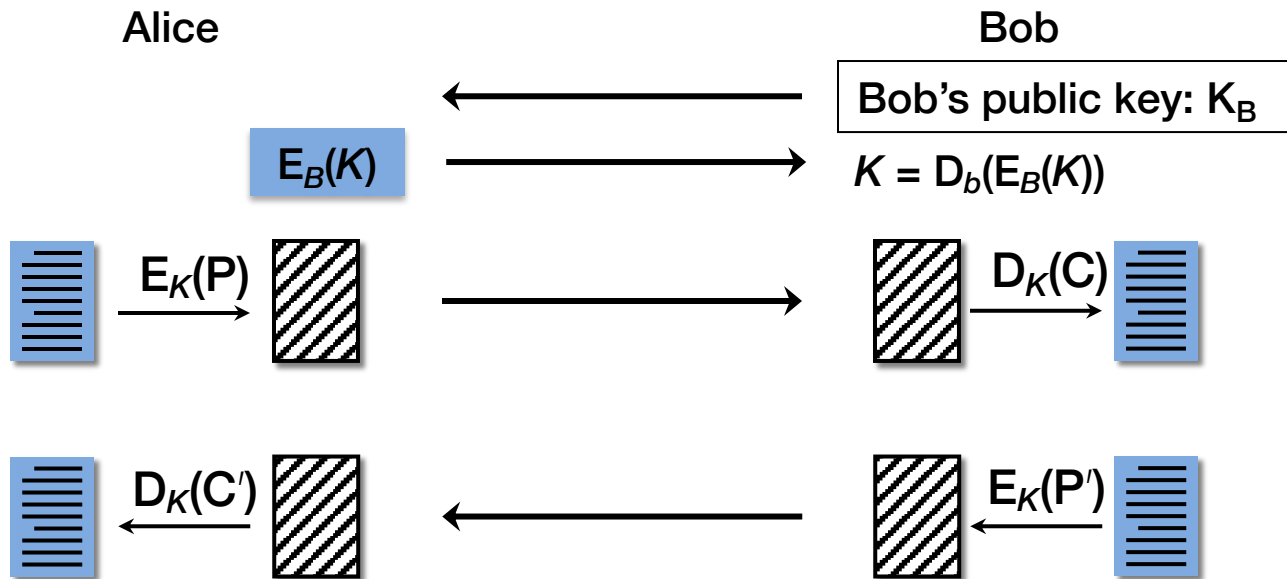
Now Bob knows the secret session key,  $K$

# Communication with a hybrid cryptosystem





# Communication with a hybrid cryptosystem



decrypt message using a symmetric algorithm and key  $K$

encrypt message using a symmetric algorithm and key  $K$

# Forward Secrecy

# Private keys need to be protected

Pick a session key & encrypt it with the Bob's public key



Bob decrypts the session key with his private key

## Suppose an attacker steals Bob's private key

- Future messages can be compromised
- The attacker can also go through past messages & decrypt old session keys

## Security rests entirely on the secrecy of Bob's private key

- If Bob's private key is compromised, all recorded past traffic can be decrypted

# Forward Secrecy

## **Forward secrecy**

- Compromise of long-term keys does not compromise past session keys
- There is no one secret to steal that will compromise multiple messages

# Achieving Forward Secrecy

Use ephemeral keys for key exchange + session keys for communication

**Diffie-Hellman key exchange is commonly used for key exchange**

- Generate a set of keys per session
- Use the derived common key as the encryption/decryption key ... or as a key to encrypt a session key
- Not recoverable as long as private keys are thrown away

Unlike RSA keys, key generation in Diffie-Hellman is extremely efficient

**Keys must be ephemeral**

Client & server will generate new Diffie-Hellman parameters for each session – all will be thrown away after the session

**Diffie-Hellman is preferred over RSA for key exchange to achieve forward secrecy.  
Generating Diffie-Hellman keys is a rapid, low-overhead process.**

# Communication with a hybrid cryptosystem (DHKE)

Alice

Bob

Create a random Diffie-Hellman key pair:  $X_A, Y_A$

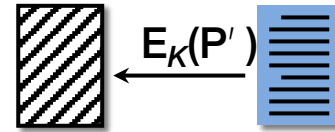
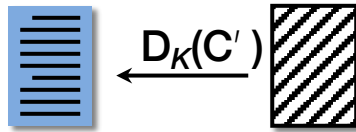
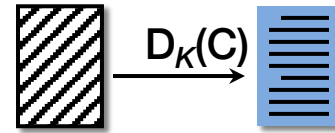
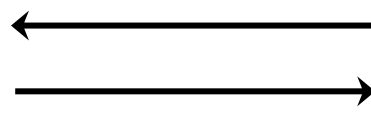
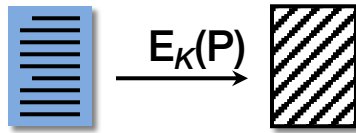
Create a random Diffie-Hellman key pair:  $X_B, Y_B$

$$K = Y_B^{X_A} \bmod p$$

Bob's D-H public key:  $Y_B$

Alice's D-H public key:  $Y_A$

$$K = Y_A^{X_B} \bmod p$$



decrypt message using a symmetric algorithm and key  $K$

encrypt message using a symmetric algorithm and key  $K$

# Cryptographic systems: summary

- **Symmetric ciphers**
  - Based on SP-networks (usually) = substitution & permutation sequences
- **Asymmetric ciphers – public key cryptosystems**
  - Based on **trapdoor** functions: easy to compute in one direction, difficult to compute in the other direction without special information (the trapdoor)
- **Hybrid cryptosystem**
  - Pick a random session key + public key algorithm for key exchange
  - Use a symmetric key algorithm to encrypt traffic back & forth
  - **Forward secrecy**: establish session key via ephemeral keys
- **Key exchange algorithms (more to come later)**
  - Diffie-Hellman
  - Public key

} *Enables secure communication without knowledge of a shared secret*
- **Perfect secrecy**
  - Ephemeral keys + Session key

# Looking ahead



# RSA cryptography in the future

- **Based on the difficulty of factoring products of two large primes**
- **Factoring algorithms get more efficient as numbers get larger**
  - As the ability to decrypt numbers increases, the key size must therefore grow even faster
  - This is not sustainable (especially for embedded devices)
- **ECC is a better choice for most applications**

# Quantum Computers & Cryptography

## **Once (if) useful quantum computers can be built, they can**

### – Factor efficiently

- Shor's algorithm factors numbers exponentially faster
- RSA will not be secure anymore

### – Find discrete logarithms & elliptic curve discrete logarithms efficiently

- Diffie-Hellman key exchange & ECC will not be secure

# Not all is bad

## **Symmetric cryptography is largely immune to attacks**

Some optimizations are predicted (Grover's algorithm): crack a symmetric cipher in time proportional to the square root of the key space size:  $2^{n/2}$  vs.  $2^n$

– Use 256-bit AES to be safe

**2016: NSA called for a migration to “post-quantum cryptographic algorithms”  
... but no agreement yet on what those will be**

**2020: Narrowed submissions down to 7 finalists & 8 alternates**

**2023: Four quantum-resistant finalists announced**

**2024: NIST post-quantum cryptographic standard expected to be finalized**

<https://csrc.nist.gov/projects/post-quantum-cryptography>

<https://www.nist.gov/news-events/news/2020/07/nists-post-quantum-cryptography-program-enters-selection-round>

# Quantum-proofing cryptography

## Quantum computing is not faster at everything

Only four types of problems are currently identified where quantum computing offers an advantage

Researchers have been developing algorithms that are be made more efficient with quantum computing



31108953	1190018662
104910828	2598220447
3027417464	3006531459
2376520867	804531264
2430217482	1122428373

Example: Add 3 out of a set of 10 numbers

- Give the sum to a friend and ask them to determine which numbers were added
- Try this if someone picks 500 out of 1,000 numbers with 1,000 digits each

Which 3 numbers add up to 5656746864?

<https://www.scientificamerican.com/article/new-encryption-system-protects-data-from-quantum-computers/>

# Stay tuned...

- **2016: NSA called for a migration to “post-quantum cryptographic algorithms”**
- **July 2020: Narrowed submissions down to 7 finalists & 8 alternates**

## **Solution families**

1. Lattice-based
2. Code-based
3. Multivariate

**Four quantum-resistant algorithms were selected**

**NIST post-quantum cryptographic standard expected to be finalized in 2024**

# Message Integrity

# McCarthy's Spy Puzzle (1958)

## The setting:

- Two countries are at war
- One country sends spies to the other country
- To return safely, spies must give the border guards a password

## Conditions

- Spies can be trusted
- Guards chat – information given to them may leak

# McCarthy's Spy Puzzle

## Challenge

- How can a border guard authenticate a person without knowing the password?
- Enemies cannot use the guard's knowledge to introduce their own spies



# Solution to McCarthy's puzzle

## Michael Rabin, 1958

- Use a **one-way function**,  $B = f(A)$ 
  - Guards get B
    - Enemy cannot compute A if they know A
  - Spies give A, guards compute  $f(A)$ 
    - If the result is B, the password is correct.
- **Example function:**
  - Middle squares
    - Take a 100-digit number (A), and square it
    - Let B = middle 100 digits of 200-digit result

# One-way functions

- Easy to compute in one direction
- Difficult to compute in the other

Examples:

## Factoring:

$$pq = N$$

find  $p, q$  given  $N$

EASY

DIFFICULT

} Basis for RSA

## Discrete Log:

$$a^b \bmod c = N$$

find  $b$  given  $a, c, N$

EASY

DIFFICULT

} Basis for Diffie-Hellman & Elliptic Curve

# Example of a one-way function: middle squares

Example with a 20-digit number

$A = 18932442986094014771$

$A^2 = 358437397421700454779607531189166182441$

Middle square,  $B = 42170045477960753118$

Given  $A$ , it is easy to compute  $B$

Given  $B$ , it is difficult to compute  $A$

“**Difficult**” = no known short-cuts; requires an exhaustive search

# Cryptographic hash functions

# Cryptographic hash functions

## Properties

- Arbitrary length input → **fixed-length output**
- **Deterministic**: you always get the same hash for the same message
- **One-way function** (**pre-image resistance**, or *hiding*)
  - Given  $H$ , it should be difficult to find  $M$  such that  $H = \text{hash}(M)$
- **Collision resistant**
  - Infeasible to find any two different strings that hash to the same value:  
Find  $M, M'$  such that  $\text{hash}(M) = \text{hash}(M')$
- **Output should not give any information about any of the input**
  - Like cryptographic algorithms, relies on *diffusion*
- **Efficient**
  - Computing a hash function should be computationally efficient

Also called *digests* or *fingerprints*

# Hash functions are the basis of integrity

- **Not encryption**
- **Can help us to detect:**
  - **Masquerading:**
    - Insertion of message from a fraudulent source
  - **Content modification:**
    - Changing the content of a message
  - **Sequence modification:**
    - Inserting, deleting, or rearranging parts of a message
  - **Replay attacks:**
    - Replaying valid sessions

# Hash Algorithms

Use iterative structure like block ciphers do ... but use no key

- **Example:**

- Secure Hash Algorithm, **SHA-1**

- Designed by the NSA in 1993; revised in 1995
- Used in the NIST Digital Signature Standard (DSS)
- Produces 160-bit hash values
- Chosen prefix collision attacks were demonstrated in May 2019

- **Successors**

- **SHA-2** (2001) – *SHA-256, SHA-384, SHA-512*

- Produces 224, 256, 384, or 512-bit hashes
- Approved for use with the NIST Digital Signature Standard (DSS)

- **SHA-3** (2015)

- Can be substituted for SHA-2
- Improved robustness

# Example: SHA-1 Overview

- **Prepare the message**

- Append the bit 1 to the message
- Pad message with 0 bits so its length =  $448 \bmod 512$
- Append length of message as a 64-bit big endian integer

- **Use an Initialization Vector (IV) = 5-word (160-bit) buffer:**

a = 0x67452301    b = 0xefcdab89    c = 0x98badcfe  
d = 0x10325476    e = 0xc3d2e1f0

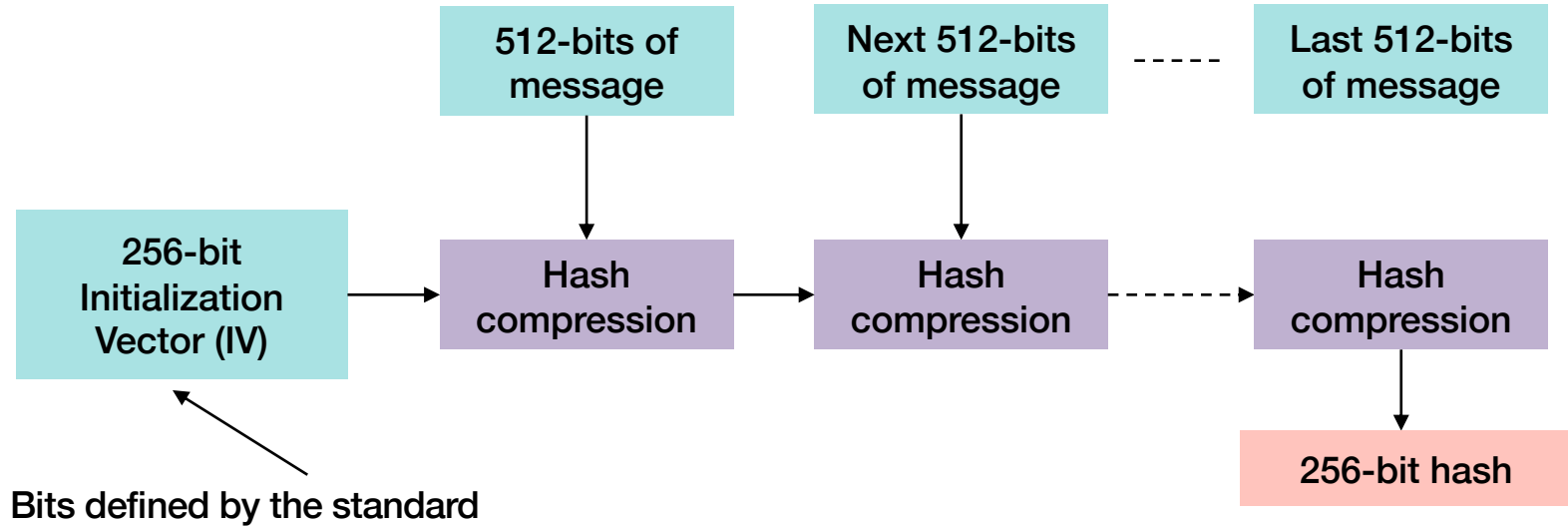
- **Process the message in 512-bit chunks**

- Expand the 16 32-bit words into 80 32-bit words via XORs & shifts
- Iterate 80 times to create a hash for this chunk
  - Various sets of ORs, XORs, ANDs, shifts, and adds
- Add this hash chunk to the result so far

See <https://www.saylor.org/site/wp-content/uploads/2012/07/SHA-1-1.pdf>



# SHA-2 Overview



# Popular (& formerly popular) Hash Functions

## MD5

- 128 bits
- Linux passwords used to use this
- Rarely used now since weaknesses were found

## SHA-1

- 160 bits – was widely used: still used as a checksum in Git & torrents
- Google demonstrated a *collision attack* in Feb 2017
  - ... Google had to run >9 quintillion SHA-1 computations to complete the attack
  - ... but already being phased out since weaknesses were found earlier
- Used for message integrity in GitHub

## SHA-2

*Believed to be secure*

- Designed by the NSA; published by NIST
- Variations: SHA-224, SHA-256, SHA-384, SHA-512
- Linux passwords use SHA-512
- Bitcoin uses SHA-256

Believed to be secure

## SHA-3

*Believed to be secure*

- 256 & 512 bit

Believed to be secure

## bcrypt

- Blowfish cipher used for *bcrypt* password hashing in OpenBSD since 1997
- Phased out in 2023: *scrypt* and *Argon2* are replacements

Designed to be slow!

## 3DES

- Linux passwords used to use this

# Creating hashes via the openssl command

## MD5 hash

```
echo 'hello, world!' | openssl dgst -md5
MD5(stdin)= 910c8bc73110b0cd1bc5d2bcae782511
```

## SHA-1 hash

```
echo 'hello, world!' | openssl dgst -sha1
SHA1(stdin)= e91ba0972b9055187fa2efa8b5c156f487a8293a
```

## 256-bit SHA-2 hash

```
echo "hello, world!" | openssl dgst -sha2-256
SHA2-256(stdin)= 4dca0fd5f424a31b03ab807cbae77eb32bf2d089eed1cee154b3afed458de0dc
```

## 256-bit SHA-3 hash

```
echo "hello, world!" | openssl dgst -sha3-256
SHA3-256(stdin)= 5208fd28810f11b7781a86289fb9121ccc754a5bd8260bcfa539163890092c7e
```

## 512-bit SHA-3 hash

```
echo "hello, world!" | openssl dgst -sha3-512
SHA3-512(stdin)=
8fc33b84ff22559082893fdc73f6877e590eb67533441fe5e48cd6d8a11aaf8d6270f82ef437c2c758000d65b09b4511
6b9c0eb3f3162149b13ca98c8cc8c90f
```

# Hash Collisions

Hashes are *collision resistant*, but collisions can occur

## Pigeonhole principle

- If you have 10 pigeons & 9 compartments, at least one compartment will have more than one pigeon
- A hash is a fixed-size small number of bits (e.g., 256 bits = 32 bytes)
- Every possible permutation of an arbitrary number of bytes cannot fit into every permutation of 32 bytes!



wikipedia

# Collisions: The Birthday Paradox

*How many people need to be in a room such that the probability that two people will have the same birthday is  $> 0.5$ ?*

*Your guess before you took a probability course: 183*

This is true to the question of “how many people need to be in a room for the probability that someone else will have the same birthday as *one specific student*?”

**Answer: 23**

$$p(n) = 1 - \frac{n! \cdot \binom{365}{n}}{365^n}$$

Approximate solution for # people required to have a 0.5 chance of a shared birthday, where  $m = \#$  days in a year

$$n \approx \sqrt{2 \times m \times 0.5}$$

# The Birthday Paradox: Implications

- **Searching for a collision with a pre-image (known message) is A LOT harder than searching for two messages that have the same hash**
- **Strength of a hash function is approximately  $\frac{1}{2}$  (# bits)**
  - 256-bit hash function has a strength of approximately 128 bits
  - But that's a huge space!  
$$2^{128} = 3.4 \times 10^{38}$$
  - It's not feasible to try that many messages in the hope of finding a collision
    - BTW ... the odds of winning the Powerball lottery are only 1:2.9 $\times 10^8$

# Message Integrity

## How do we detect that a message has been tampered?

- A cryptographic hash acts as a checksum
- Associate a hash with a message
  - We're not encrypting the message
  - We're concerned with *integrity*, not *confidentiality*
- If two messages hash to different values, we are convinced that the messages are different

$$H(M) \neq H(M')$$

# Tamperproof Integrity: Message Authentication Codes and Digital Signatures



# MACs (also called a Keyed Hash)

We rely on hashes to assert the integrity of messages

But an attacker can create a new message & a new hash  
and replace  $H(M)$  with  $H(M')$

So, let's create a checksum that relies on a key for validation

**Message Authentication Code (MAC)**

Two forms: **hash-based** & **block cipher-based**

# HMAC: Hash-based MAC

We can create a MAC from a cryptographic hash function

**HMAC = Hash-based Message Authentication Code**

$$HMAC(m, k) = H((opad \oplus k) \parallel H(ipad \oplus k) \parallel m))$$

where

$H$  = cryptographic hash function

$opad$  = outer padding 0x5c5c5c5c ... (01011100...)

$ipad$  = inner padding 0x36363636... (00110110...)

$k$  = secret key

$m$  = message

$\oplus$  = XOR,  $\parallel$  = concatenation

Note the extra hash.

The simple form of an HMAC would simply be  $hash(m, k)$

The HMAC standard devised this to strengthen the HMAC against weaker hash functions.

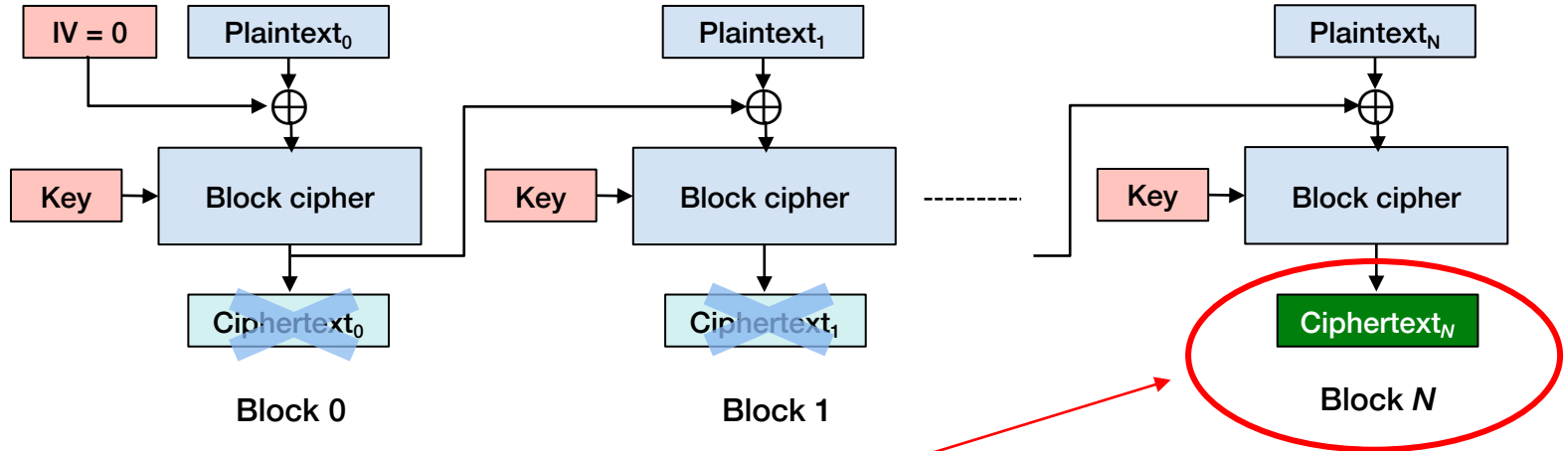
**Basically, incorporate a key into the message before hashing it**

See RFC 2104

# Block Cipher Based MAC: CBC-MAC

**Cipher Block Chaining (CBC)** ensures that every encrypted block is a function of all previous blocks

CBC-MAC uses an initialization vector = 0



**MAC = final ciphertext block – others are discarded**

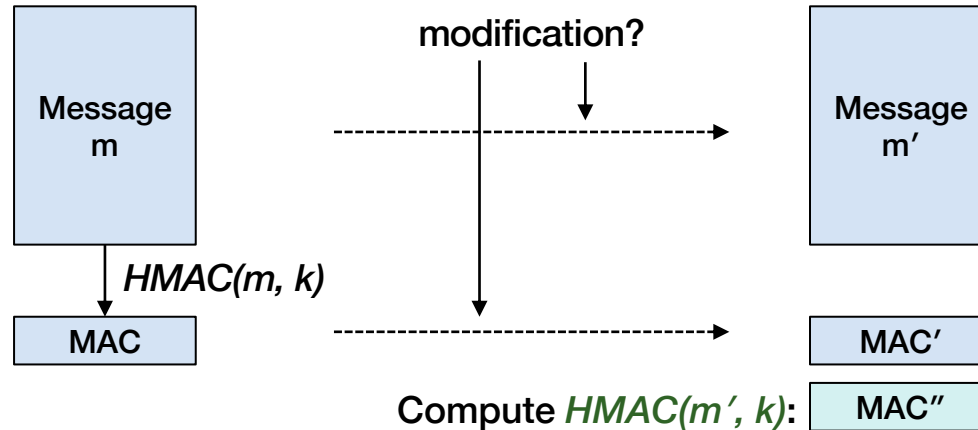
**Examples: AES-CBC-MAC, DES-MAC**

**Don't use the same key for the MAC as for encrypting the message**

If an adversary gets one of the keys, she will be unable to create either a valid message or a valid hash

# Using a MAC

Alice ← Both have the shared key,  $k$  → Bob



1. Bob receives the Message  $m'$  and a MAC.
2. Knowing the key,  $k$ , he generates a MAC for the message:  $MAC'' = HMAC(m', k)$
3. If  $MAC' = MAC''$ , he's convinced that the message has not been modified

# Digital Signatures

- **MACs rely on a shared key**
  - Anyone with the key can modify and re-sign a message
- **Digital signature properties**
  - Only you can sign a message, but anyone can validate it
  - You cannot cut and paste the signature from one message to another
  - If the message is modified, the signature will be invalid
  - An adversary cannot forge a signature
    - Even after inspecting an arbitrary number of signed messages

# Digital Signature Primitives

## 1. Key generation

$\{ \text{secret\_key}, \text{verification\_key} \} := \text{gen\_keys}(\text{key\_size})$

## 2. Signing

$\text{signature} := \text{sign}(\text{message}, \text{secret\_key})$

## 3. Validation

$\text{Isvalid} := \text{verify}(\text{verification\_key}, \text{message}, \text{signature})$

We sign *hash(message)* instead of the *message*

- We'd like the signature to be a small, fixed size
- We may not need to hide the contents of the message
- We trust hashes to be collision-free

# Digital Signatures & Public Key Cryptography

**Public key cryptography enables digital signatures**

*secret\_key = private key*

*verification\_key = public key*

- Alice encrypts a message with her **private** key

$$S = E_a(M)$$

- Anyone can decrypt it using her **public** key

$$D_A(S) = D_A(E_a(M)) = M$$

- Nobody but Alice can create S

# Popular Digital Signature Algorithms

**Digital Signature Algorithms combine hashing + encryption into one step**

**signature:  $S := E_{pri\_key}(H(M))$**

**verification =  $H(M) \stackrel{?}{=} D_{pub\_key}(S)$**

- **DSA: Digital Signature Algorithm**
  - NIST standard – Uses SHA-1 or SHA-2 hash
  - Key pair based on difficulty of computing discrete logarithms
- **ECDSA: Elliptic Curve Digital Signature Algorithm**
  - Variants of DSA that uses elliptic curve cryptography
  - Used in bitcoin
- **EdDSA: Edwards-curve Digital Signature Algorithm**
  - Slightly faster than ECDSA



# Digital signatures



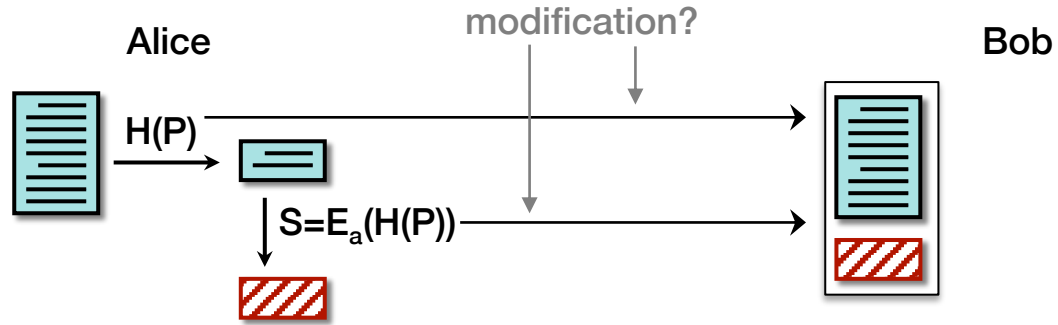
Alice generates a hash of the message,  $H(P)$

# Digital signatures: public key cryptography



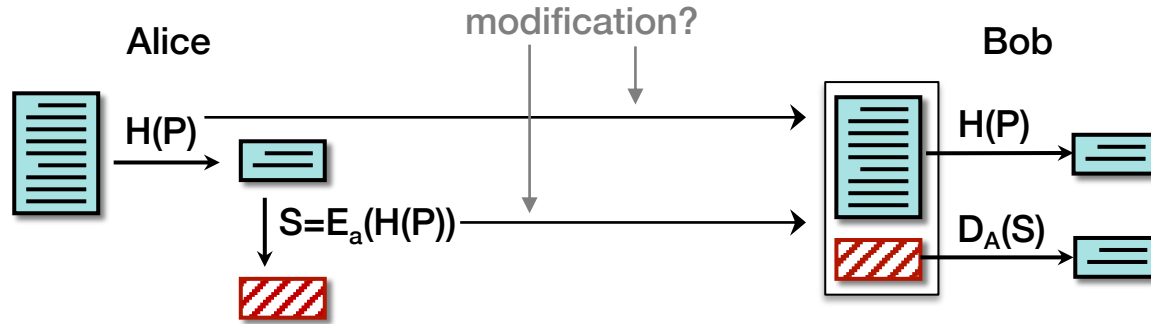
Alice encrypts the hash with her private key  
This is her **signature**.

# Using Digital Signatures



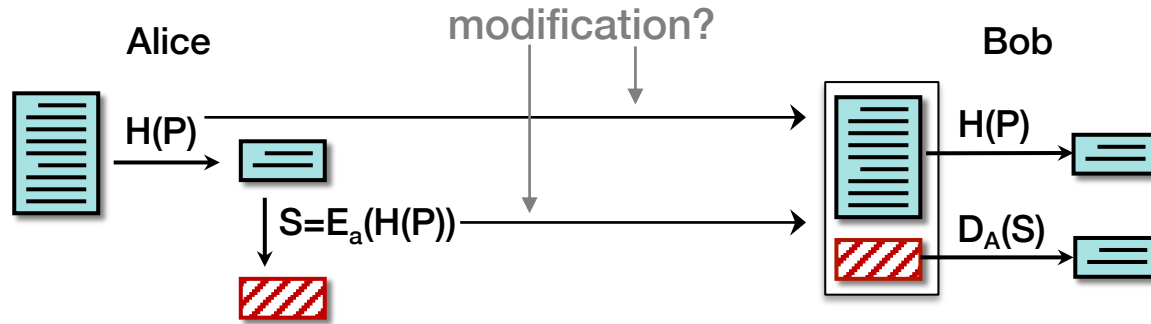
Alice sends Bob the message & the encrypted hash

# Using Digital Signatures



1. Bob decrypts the hash using Alice's **public key**
2. Bob computes the hash of the message sent by Alice

# Using Digital Signatures

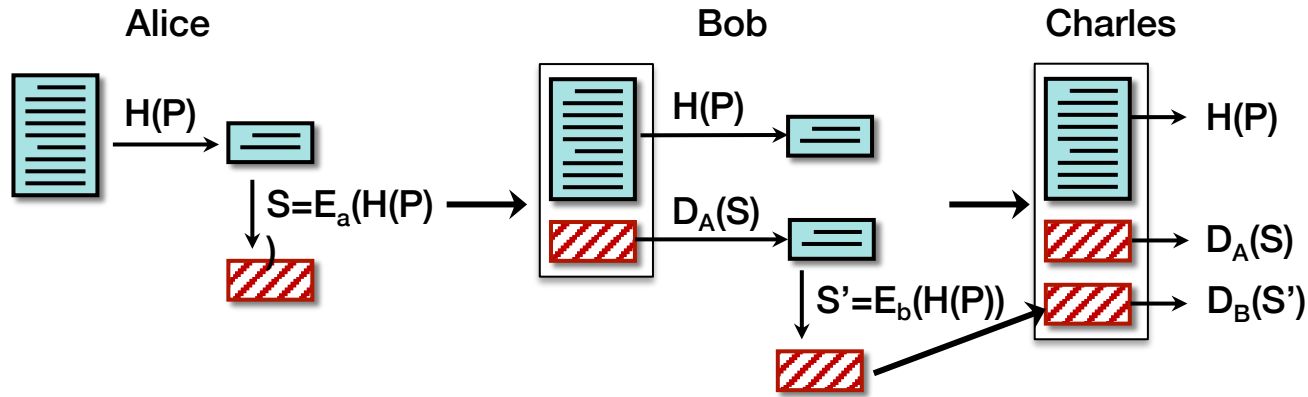


If the hashes match, the signature is valid  
⇒ the encrypted hash *must* have been generated by Alice

# Digital signatures & non-repudiation

- **Digital signatures provide non-repudiation**
  - Only Alice could have created the signature because only Alice has her private key
- **Proof of integrity**
  - The hash assures us that the original message has not been modified
  - The encryption of the hash assures us that an attacker could not have re-created the hash

# Digital signatures: multiple signers



Charles:

- Generates a hash of the message,  $H(P)$
- Decrypts Alice's signature with Alice's public key
  - Validates the signature:  $D_A(S) \stackrel{?}{=} H(P)$
- Decrypts Bob's signature with Bob's public key
  - Validates the signature:  $D_B(S) \stackrel{?}{=} H(P)$

# Covert AND authenticated messaging

## If we want to keep the message secret

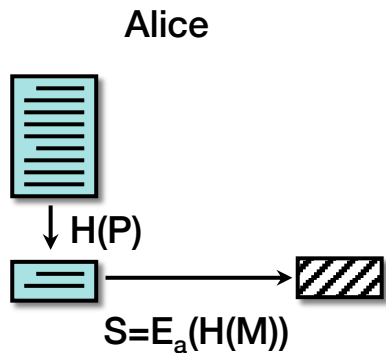
- combine **encryption** with a **digital signature**

## Use a session key:

- Pick a **random key,  $K$** , to encrypt the message with a symmetric algorithm
- **Encrypt  $K$**  with the public key of each recipient
- For signing, **encrypt the hash** of the message with sender's private key

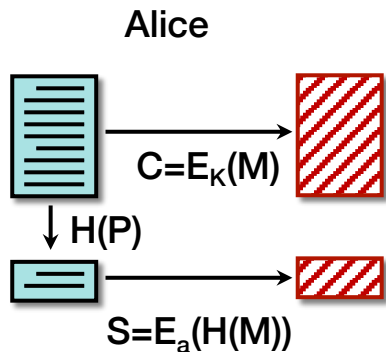


# Covert and authenticated messaging



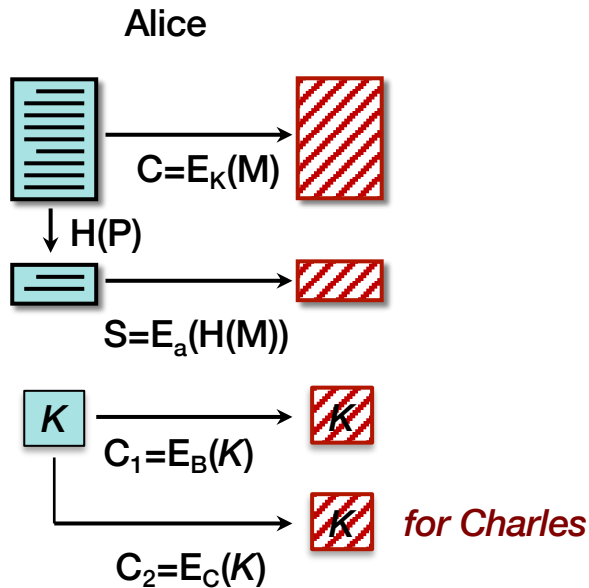
**Alice generates a digital signature by encrypting the message with her private key**

# Covert and authenticated messaging



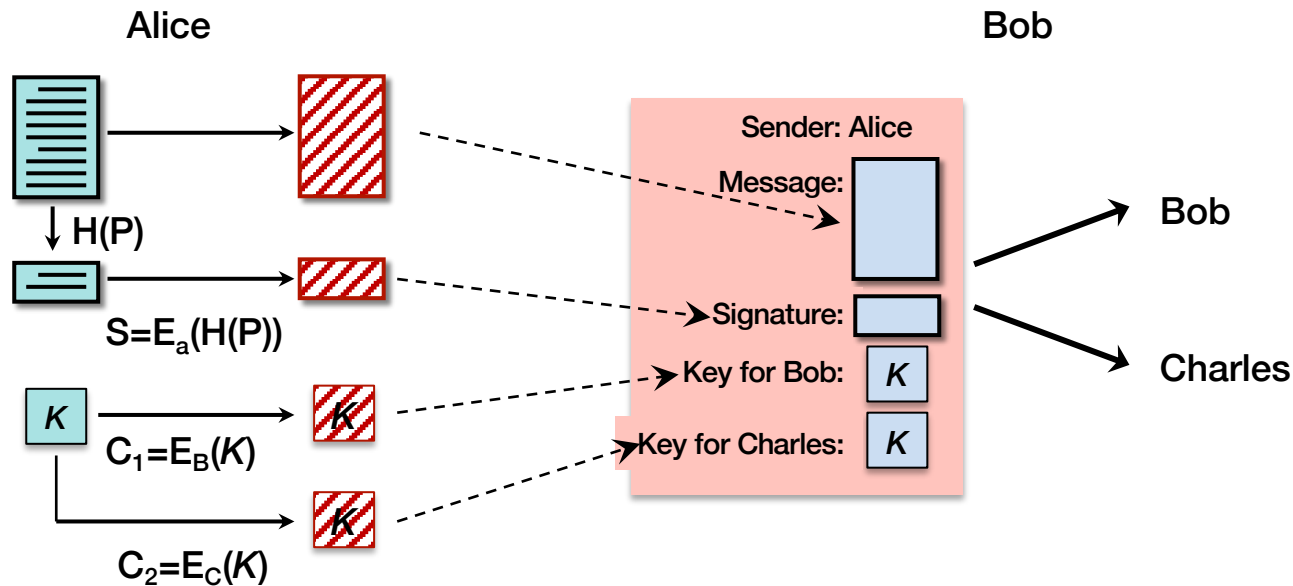
Alice picks a random key,  $K$ , and encrypts the message  $P$  with it using a symmetric cipher

# Covert and authenticated messaging



**Alice encrypts the session key for each recipient of this message using their public keys**

# Covert and authenticated messaging



**The aggregate message is sent to Bob & Charles**

**Note: we do not have forward secrecy by doing this**

# Certificates: Identity Binding

# Public Keys as Identities

- **A public signature verification key can be treated as an identity**
  - Only the owner of the corresponding private key will be able to create the signature
- **New identities can be created by generating new random {private, public} key pairs**
- **Anonymous identity – no identity management**
  - A user is known by a random-looking public key
  - Anybody can create a new identity at any time
  - Anybody can create as many identities as they want
  - A user can throw away an identity when it is no longer needed
  - Example: your Bitcoin identity = hash(public key)

# Identity Binding

- **How does Alice know Bob's public key is really his?**
- **Get it from a trusted server?**
  - What if the enemy tampers with the server?
  - Or intercepts Alice's query to the server (or the reply)?
  - What set of public keys does the server manage?
  - How do you find it in a trustworthy manner?

# Identity Binding – Another Option

- **Have a trusted party sign Bob's public key**
- **Once signed, it is tamper-proof**
  - An attacker cannot generate the signature after modifying the key
- **But we need to know it's Bob's public key and who signed it**
  - Create & sign a data structure that
    - Identifies Bob
    - Contains his public key
    - Identifies who is doing the signing

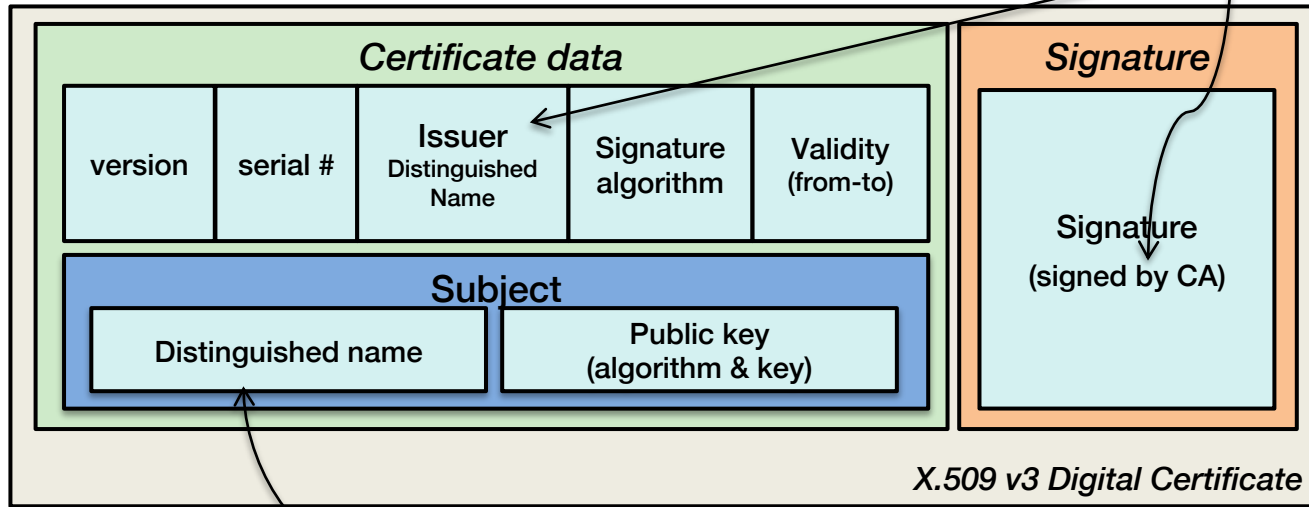


# X.509 Certificates

ISO introduced a set of authentication protocols

X.509: Structure for public key certificates:

Issuer = **Certification Authority (CA)**



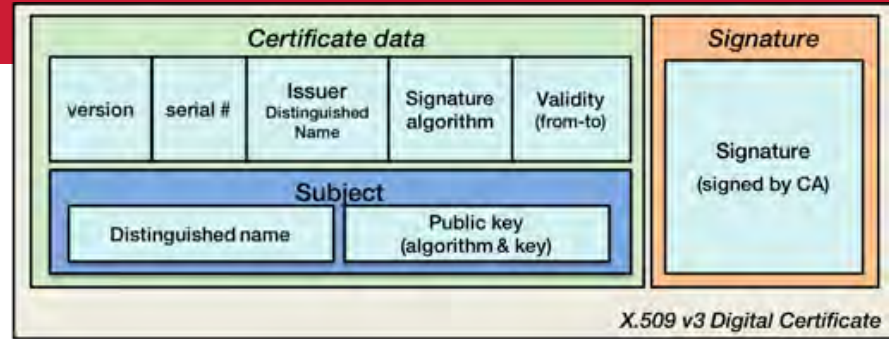
*User's name, organization, locality, state, country, etc.*

# X.509 Certificates

## To validate a certificate

Verify its signature:

1. Get the issuer (CA) from the certificate
2. Validate the certificate's signature against the issuer's public key
  - Hash contents of certificate data
  - Decrypt CA's signature with CA's public key



Obtain CA's public key (certificate) from trusted source

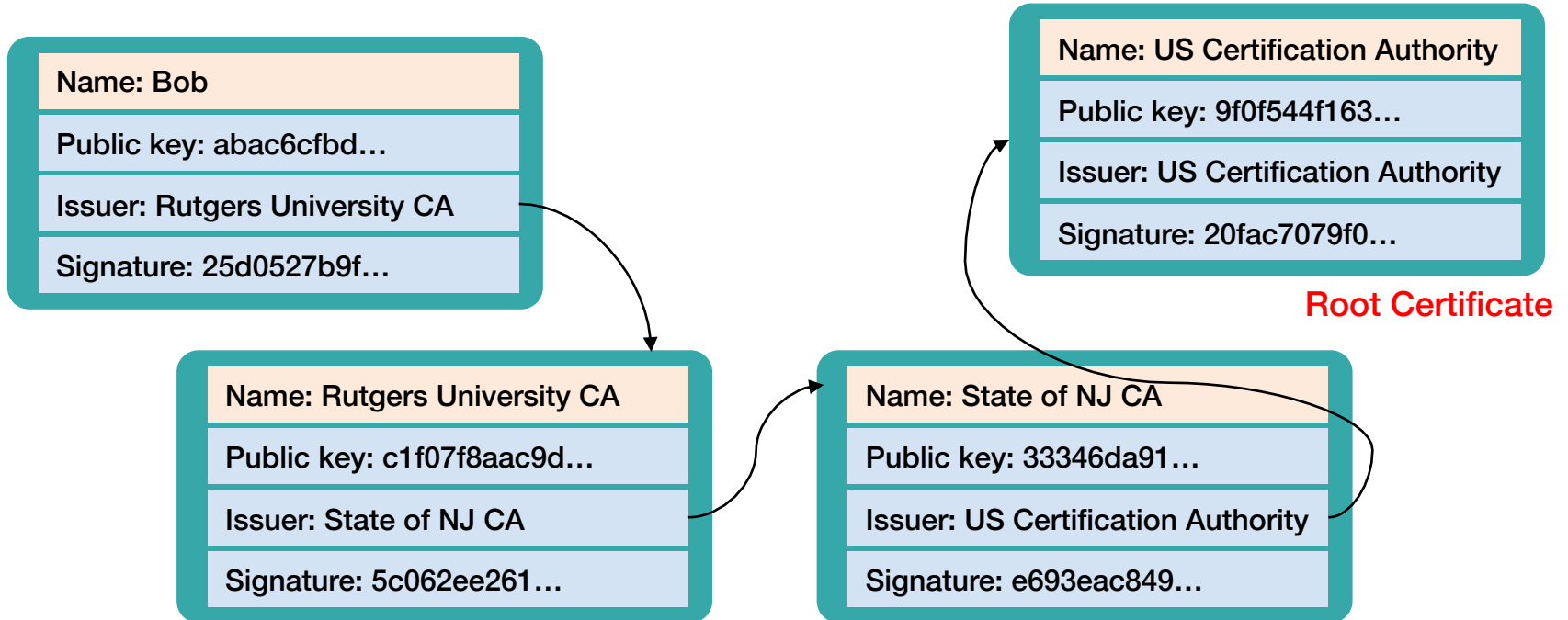
**Certificates prevent someone from using a phony public key to masquerade as another person**

***...if you trust the CA***

# Certification Authorities (CAs)

## How do you know the public key of the CA?

- You can get it from another certificate! ⇒ this is called **certificate chaining**



# Certification Authorities (CAs)

- **But trust must start somewhere**

**You need a public key you can trust – this is the root certificate**

- **Apple's Trust Store** is pre-loaded with over 160 CA certificates
  - Stores non-personal security info; accessed via Keychain
- Windows stores them in the Certificate Store and makes them accessible via the **Microsoft Management Console** (mmc)
- Android stores them in **Credential Storage**

- **Can you trust a CA?**

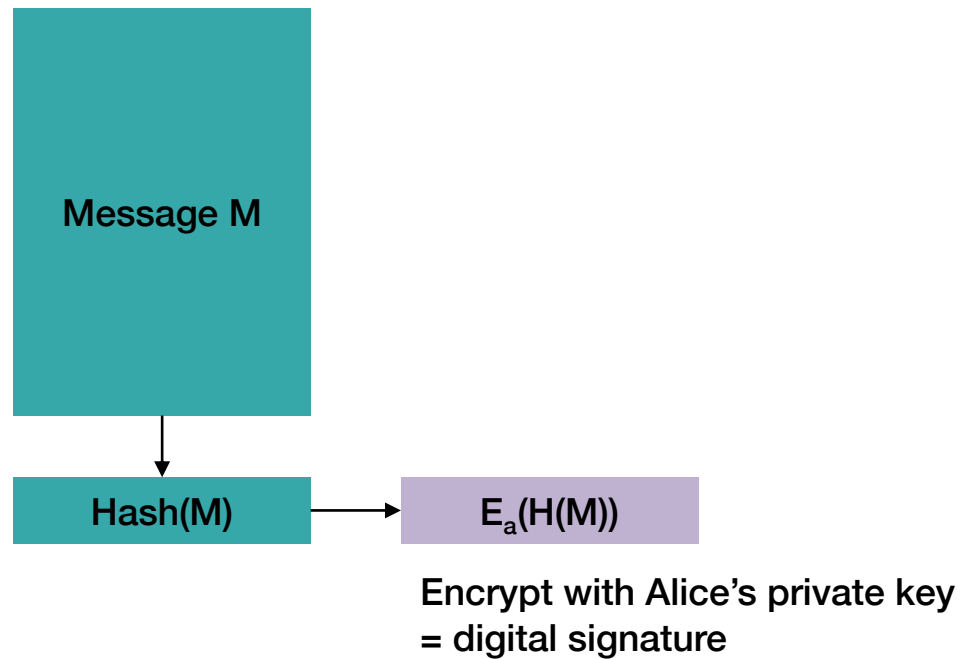
- Maybe...  
check their reputation and read their **Certification Practice Statement** (CPS)
- Even trustworthy ones might get hacked (e.g., VeriSign in 2010)

# Key revocation

- **Used to invalidate certificates before expiration time**
  - Usually because of a compromised key
  - Or policy changes (e.g., someone leaves a company)
- **Certificate revocation list (CRL)**
  - Lists certificates that are revoked
  - Only certificate issuer can revoke a certificate
- **Problems**
  - Need to make sure that the entity issuing the revocation is authorized to do this
  - Revocation information may not circulate quickly enough
    - Dependent on dissemination mechanisms, network delays & infrastructure
  - Some systems may not have been coded to process revocations

# Code Integrity

# Review: signed messages



# We can sign code as well

- **Validate integrity of the code**
  - If the signature matches, then the code has not been modified
- **Enables**
  - Distribution from untrusted sources
  - Distribution over untrusted channels
  - Detection of modifications by malware
- **Signature = encrypted hash signed by trusted source**
  - Does not validate the code is good ... just where it comes from



# Code Integrity: **signed software**

- **Windows since XP\*: Microsoft Authenticode**
  - *SignTool* command
  - Hashes stored in system catalog or signed & embedded in the file
  - Microsoft-tested drivers are signed
- **macOS**
  - *codesign* command
  - Hashes & certificate chain stored in file
- **Also Android & iOS**

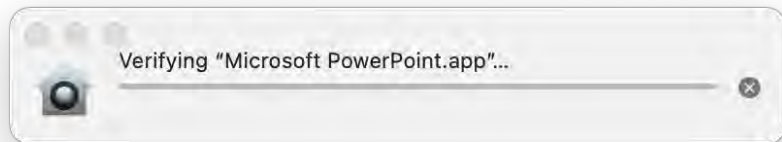
\*Windows XP had partial support for Authenticode; it did not support signed drivers.

# Code signing: Microsoft Authenticode

- **A format for signing executable code (dll, exe, cab, ocx, class files)**
- **Software publisher:**
  - Generate a public/private key pair
  - Get a digital certificate from a certification authority (CA) that is enrolled in the *Microsoft Trusted Root Certificate Program*
  - Generate a hash of the code to create a fixed-length digest
  - Encrypt the hash with your private key
  - Combine digest & certificate into a Signature Block
  - Embed Signature Block in executable package
- **Microsoft SmartScreen:**
  - Manages reputation based on download history, popularity, anti-virus results
- **Recipient:**
  - Call *WinVerifyTrust* function to validate:
    - Validate certificate, decrypt digest, compare with hash of downloaded code

# Per-page hashes

- **Integrity check when program is first loaded (this takes time)**



- **Check a hash for a page when it is needed (demand paging)**
  - This is efficient (pages are small; checking a hash is quick)

Per-page hashes can be disabled optionally on both Windows and macOS

# Windows code integrity checks

- **Implemented as a file system driver**
  - Works with demand paging from executable
  - Check hashes for every page as the page is loaded
- **Hashes stored in system catalog or embedded in file along with X.509 certificate**
- **Check integrity of boot process**
  - Kernel code must be signed or it won't load
  - Drivers shipped with Windows must be certified or contain a certificate from Microsoft

The End